# Lesson 18: Projectile Motion at an Angle

To do questions involving objects launched from the ground upwards at an angle (like kicking a football up into the air and watching it as it arcs in the air and comes back down), you need to add a few more steps to the way you did the **questions** for objects launched **horizontally**.

- There are actually two ways to do these types of problems, one based on the **vertical velocity** of the object, the other based on the **vertical displacement**.
    - The only big difference in these methods is how we are going to calculate the time that the object spends in the air.
- Choose whichever method you are most comfortable with, and whichever one suits the particular question you are doing. We will look at each and then it's up to you to figure out which way you will approach a problem.

Imagine for a moment that you are watching an object as it rises into the air after you kick it upwards at an angle. Look at Illustration 1 below as you read through this description.
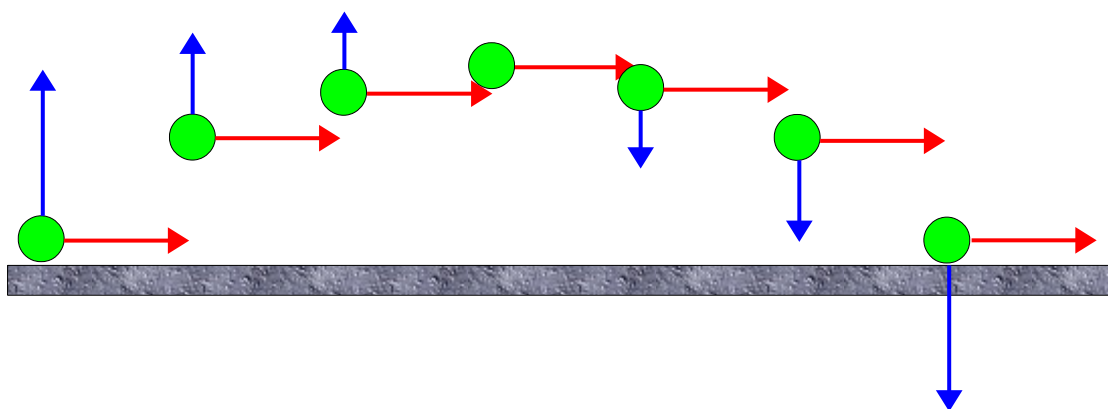
Illustration 1: Object launched at an angle.

- When it left your foot, it was going at the fastest that it can possibly move during its flight.
- The instant it leaves your foot, gravity is pulling down on it, causing it to have less and less **vertical velocity**.
- Remember that there will be no change in the **horizontal component** of its velocity.
- When it reaches the highest point in its flight, it isn't moving up, and it isn't moving down, for an instant of time… its **vertical velocity** is ZERO!
- By the time it reaches the ground again, it will still be moving with its original **horizontal velocity** and will have just as much **vertical velocity** as when it left your foot. It will have the exact same velocity as it left your foot with!

We can use this information about its **vertical movement** to do some calculations. We know...

- that there is gravity (-9.81m/s^2) causing the acceleration on the object **vertically.**
- the initial **vertical velocity** of the object.
- the final **vertical velocity** of the object. We can even use this two ways, since we can say that the final **vertical velocity** happens at the halfway point (zero m/s), or when it gets back to the ground (same as it left the ground at).

This gives us enough information to calculate the maximum height of the flight, and the time it spends in the air. After that, we can calculate just about anything…

Example 1: You kick a soccer ball at an angle of 40° above the ground with a velocity of 20m/s. **Determine**...
   a) How high will it go?
   b) How much time does it spend in the air?
   c) How far away from you will it hit the ground (aka *range*)?
   d) What is the ball's velocity when it hits the ground?

Before we can calculate anything else, we first need to break the original velocity into components.
   - We do this so we have a **vertical component** to do the first couple calculations with. The **horizontal component** will be used later.
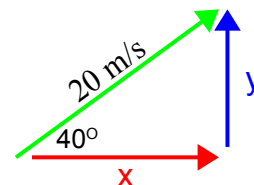
$$Y-Component$$
$$\sin\theta=\frac{opp}{hyp}$$
$$opp=\sin\theta(hyp)$$
$$opp=\sin40^O(20)$$
$$opp=12.85575219=13m/s$$

Illustration 2: Components of original trajectory.

$$X-Component$$
$$\cos\theta=\frac{adj}{hyp}$$
$$adj=\cos\theta(hyp)$$
$$adj=\cos40^O(20)$$
$$adj=15.3208889=15m/s$$

a) **THINK VERTICAL!**
   At its maximum height, halfways through its flight, the object won't be going up or down, so we'll say that its final velocity at that point is zero.
$$v_f^2=v_i^2+2ad$$
$$d=\frac{(v_f^2-V_i^2)}{2a}$$
$$d=\frac{0^2-12.85575219^2}{2(-9.81)}$$
$$d=8.42356597=8.4\,m$$

The ball will reach a maximum height of 8.4 m.

b) **THINK VERTICAL!**
   The following three methods show slightly different ways of approaching the problem.
   - In the first, we will assume that when the ball strikes the ground it is traveling at the same **velocity** down as it originally started at up.
   - In the second, we will only calculate the time it gets to the halfway point, at exactly the highest point it reaches. At that point the vertical **velocity** is zero. This just means you have to double your final answer to get the whole time it

was in the air.
- In the third method we will assume that the ball strikes the ground at the same height that it left the ground from. This means that it has a **vertical displacement** of zero.
- Note: I used the unrounded value of 12.85575219m/s in each of these calculations, but just wrote 13m/s to save space.

Method One

$$a = \frac{v_f - v_i}{t}$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{-13 - 13}{-9.81}$$

$$t = 2.6s$$

Method Two

$$a = \frac{v_f - v_i}{t}$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{0 - 13}{-9.81}$$

$$t = 1.3s$$

Multiplied by two = 2.6s

Method Three

$$d = v_i t + 1/2\, at^2$$

$$0 = v_i t + 1/2\, at^2$$

$$-v_i t = 1/2\, at^2$$

$$-v_i = 1/2\, at$$

$$t = \frac{-2v_i}{a}$$

$$t = \frac{-2(13)}{-9.81} = 2.6s$$

The unrounded answer for each method shown here is 2.620948459 s.

c) **THINK HORIZONTAL**

It is moving at a **constant velocity horizontally** during the whole time we just figured out, so let's use the **horizontal component** of the velocity to figure out the **displacement horizontally**.

$$v = \frac{d}{t}$$

$$d = vt$$

$$d = 15.3208889(2.620948459)$$

$$d = 40.15526015 = 40\,m$$

The **horizontal displacement** (*range*) is 40m.

d) The ball's velocity when it hits the ground is exactly the same as when it was originally launched… 20 m/s at 40° up from the horizontal. The only difference is that now it's spiking into the ground.